Reading, Writing, and Proving (Second Edition)

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Solutions to Chapter 7: Operations on Sets

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Solution to Problem 7.3. *Proof.* We note that $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$. According to Part 18 of Theorem 7.4, this is the case if and only if $(X \setminus B) \subseteq (X \setminus A)$ and $(X \setminus A) \subseteq (X \setminus B)$. This in turn is the case if and only if $(X \setminus A) = (X \setminus B)$ □

Solution to Problem 7.6. *The Venn diagram for $A \triangle B$ is in Figure 7.1*

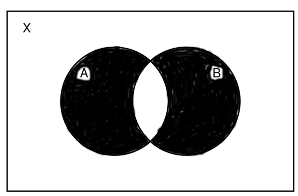

Figure 7.1: $A \triangle B$

Note that By part 17 of Theorem 7.4 we have $B \setminus A = B \cap A^c$. Then the required proof is the content of Problem 7.5 above.

Solution to Problem 7.9. *First note that $A \cap B \subseteq A \subseteq A \cup B$ by Theorem 7.4 (11) and (12). Thus $(A \cap B) \setminus (A \cup B) = \emptyset$. This will justify the second equality below:*

$$\begin{aligned}(A \cup B) \triangle (A \cap B) &= ((A \cup B) \setminus (A \cap B)) \cup ((A \cap B) \setminus (A \cup B)) \\ &= (A \cup B) \setminus (A \cap B) \\ &= A \triangle B\end{aligned}$$

The last equality was proven in Problem 7.6.

Solution to Problem 7.12. *(a) The notation for the set in* (ii) *is ambiguous.*

(b) The sets (iii), (iv), *and* (v) *are represented by the shaded area in Figure 7.1.*

(c) This is the statement of Part 15 of Theorem 7.4 (DeMorgan's law).

Solution to Problem 7.15. *First assume that* $A^c \cup B^c = X$. *Then* $\emptyset = X^c = (A^c \cup B^c)^c = ((A \cap B)^c)^c = A \cap B$. *Thus* A *and* B *are disjoint. We used Theorem 7.4 (2) and (16).*

For the converse we assume that $A \cap B = \emptyset$. *Then* $A^c \cup B^c = (A \cap B)^c = \emptyset^c = X$, *again using Theorem 7.4 (16). Thus* $A^c \cup B^c = X$.

Solution to Problem 7.18. *This statement is true and we will prove it below.*

Proof. Since we assume that $A \cap Y = B \cap Y$ for all $Y \subseteq X$, we may choose $Y = B$. Then $A \cap B = B \cap B = B$. From part 22 of Theorem 7.4 we conclude that $B \subseteq A$. Choosing $Y = A$, the same argument shows that $A \subseteq B$. Hence $A = B$. $\square$