Reading, Writing, and Proving (Second Edition)

Ulrich Daepp and Pamela Gorkin
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Solutions to Chapter 2: Logically Speaking

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Ueli Daepp and Pam Gorkin

Solution to Problem 2.3.

$\mathbf{P}$	$\mathbf{Q}$	$\neg(\mathbf{P} \wedge \mathbf{Q})$
T	T	F
T	F	T
F	T	T
F	F	T

$\mathbf{P}$	$\mathbf{Q}$	$\neg\mathbf{P} \vee \neg\mathbf{Q}$
T	T	F
T	F	T
F	T	T
F	F	T

Since both statement forms have the same truth table, we conclude from Theorem 2.7 that the two forms are equivalent; that is, $\neg(P \wedge Q) \leftrightarrow \neg P \vee \neg Q$.

Solution to Problem 2.6.

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{R}$	$\neg\mathbf{R} \vee \mathbf{Q}$	$\mathbf{P} \to (\neg\mathbf{R} \vee \mathbf{Q})$	$(\mathbf{P} \to (\neg\mathbf{R} \vee \mathbf{Q}) \wedge \mathbf{R}$
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	F	F	F
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	F	T	T
F	F	F	T	T	F

This statement form is not a tautology and it is not a contradiction since the truth table contains both values, T and F.

Solution to Problem 2.9. *(a)*

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{R}$	$\mathbf{P \wedge \neg Q}$	$\mathbf{(P \wedge \neg Q) \rightarrow R}$
T	T	T	F	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

.

(b) *By Theorem 2.6 we need a statement form that has the same truth table as the one above. Using the fact that a conjunction has only one case in which the form is true, we conclude that one possible example is*

$$\neg(P \wedge (\neg Q) \wedge \neg R).$$

(Note that there are many other possible solutions.)

Solution to Problem 2.12. *The two possibilities are $(P \wedge Q) \vee R$ and $P \wedge (Q \vee R)$. We find both truth tables to decide which statement form corresponds to the given table.*

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{R}$	$\mathbf{(P \wedge Q) \vee R}$	$\mathbf{P \wedge (Q \vee R)}$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	T	F
F	F	F	F	F

We see that the given truth table is the one of the statement form $(P \wedge Q) \vee R$.

Solution to Problem 2.15. *We will use S for "it snows," and T for "it is sunny." With this notation the form of the given statement is $S \vee \neg T$.*

(a) *From Exercise 2.8 we know that $S \vee \neg T$ is equivalent to $T \rightarrow S$. Hence an equivalent statement is "If it is sunny, then it snows."*

(b) *According to DeMorgan's laws $\neg(S \vee \neg T)$ is equivalent to $\neg S \wedge T$. Hence a negation of the given statement is "It does not snow and it is sunny."*

Solution to Problem 2.18. *We denote by A the statement "A did it." Statements B and C are defined analogously. The students' statement forms were then: $\neg B \to C$, $(A \wedge C) \vee C$, and $A \wedge B \wedge C$. It will be enough to look at the truth table that contains exactly one* T *among A, B, and C since in both cases we know that exactly one person committed the crime.*

A	**B**	**C**	$\neg\mathbf{B} \to \mathbf{C}$	$(\mathbf{A} \wedge \mathbf{C}) \vee \mathbf{C}$	$\mathbf{A} \wedge \mathbf{B} \wedge \mathbf{C}$
T	*F*	*F*	*F*	*F*	*F*
F	*T*	*F*	*T*	*F*	*F*
F	*F*	*T*	*T*	*T*	*F*

.

(a) Consulting the truth table above we conclude that Charlotte did it.

(b) Again looking at the truth table we conclude that Alan is guilty.

Solution to Problem 2.21. *(a) Setting P for "Peter ran in the race" and Q for "Paul ran in the race," we interpret statement 1 as $P \vee Q$. Note that the alternate interpretation, $(P \vee Q) \wedge \neg(P \wedge Q)$ is also possible.*

Here we denote with P: "You are with us" and with Q: "You are against us." This intent of this quote (of former President George W. Bush) was $(P \vee Q) \wedge \neg(P \wedge Q)$.

(b) An example of the possibility of both options: "I will either see my mother or I will see the whole family."

An example for only one option to occur is: "I will either spend my 22nd birthday in Spain or in Argentina."

(c) $P \dot{\vee} Q \leftrightarrow ((P \vee Q) \wedge \neg(P \wedge Q))$.

(d) The statement form of "neither P nor Q" is then $\neg(P \vee Q)$ which is equivalent to $\neg P \wedge \neg Q$. This leads to the truth table below.

P	**Q**	***neither* P *nor* Q**
T	*T*	*F*
T	*F*	*F*
F	*T*	*F*
F	*F*	*T*

.

An example of an English sentence is: "He will neither be paid for completing this task nor given days off to compensate for the work."