CORRECTIONS AND SUGGESTIONS: READING, WRITING, AND PROVING

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Note, a second edition of this text has appeared (2011).

P. 17, Line -3:
Replace sentence by: A **statement** is a sentence that is true or false but not both.

P. 22
One should include a discussion or examples that shows that P and Q are equivalent if and only if they have the same truth tables.

P. 31, Chapter 3:
Before doing Exercise 3.4, it would be a good idea to do an exercise with words.

Suggestion: If the sky is green, then $2 + 2 = 4$.
State the converse and the contrapositive.
Is the statement true?
Is the converse true?
Is the contrapositive true?

P. 37, Problem 3.7 part (d):
Change to "Find the negation of the *original* statement by writing the sentence in symbols, negating it, and then rewriting the sentence in words."

P. 41
A discussion of how a statement $p(x)$ is changed if the universe is restricted to a subset A should appear at this point. We also note that the form depends on the quantifier:

$$\forall x(x \in A \to p(x)) \qquad \text{and} \qquad \exists x(x \in A \wedge p(x)).$$

P. 43, Example 4.3
Replace "should" with "do".

P. 45, line 11:
Strike the word "either" (we did not discuss "either" in formal language).

P. 47, Problem 4.5
Change the sentence preceding part (a) to "State the universe, if appropriate."

P. 49, Problem 4.10
Replace the first sentence by: "State the negation of the statement, the converse, the negation of the converse, the contrapositive, and the negation of the contrapositive."

P. 50, Additional problem:

Let x be an integer. Consider the statement: "If 8 does not divide $x^2 - 1$, then x is even."

(a) State the universe.
(b) Write the statement in symbols.
(c) Negate the statement.
(d) Prove the statement.

P. 53, Chapter 5
Include a discussion of proofs using "if and only if" before assigning problem 5.9.

Suggestion: Students are familiar with proofs in cases for things like "For $x \in \mathbb{R}$ show that $\frac{2x+1}{x-1} \leq 1$ iff $-2 \leq x < 1$. This might be a useful example for them.

P. 60, Problem 5.9
It may be worthwhile to add a hint to (c): "Hint: use parts (a) and (b)." (Experience shows that students usually do not recognize the connection between these.)

P. 61 Additional problems:

(1) For $z, w \in \mathbb{R}$, show that $|(1+z)(1+w) - 1| \leq (1+|z|)(1+|w|) - 1$.
(2) Prove that $x^2 - y^2 = 10$ has no positive integer solutions.
(3) Prove that $b^2 + b + 1 = a^2$ has no positive integer solutions.

P. 77, Problem 6.18
Change the second question to: "Is one of the two sets contained in the other? Justify your answer."

P. 79, Chapter 7
Before assigning problems, students should be able to work the following: If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

P. 89, Chapter 8
Suggestion for additional problems in Chapter 8:

Problem. Prove that if I is a set and $\{A_\alpha\}_{\alpha \in I}$, $\{B_\alpha\}_{\alpha \in I}$ are families of indexed sets such that $A_\alpha \subseteq B_\alpha$ for all $\alpha \in I$, then $\bigcup_{\alpha \in I} A_\alpha \subseteq \bigcup_{\alpha \in I} B_\alpha$.

Problem. If I and J are index sets with $J \subseteq I$ and $\{A_\alpha\}_{\alpha \in I}$ is a family of sets, prove that

$$\bigcup_{\alpha \in J} A_\alpha \subseteq \bigcup_{\alpha \in I} A_\alpha.$$

P. 93, Exercise 8.9
Assume the index set is nonempty.

P. 95 Problem 8.7 Replace (a) and (b) by adding sub-subscripts:

(a) If $A_{\alpha_0} = \emptyset$ for some $\alpha_0 \in I$, prove that $\bigcap_{\alpha \in I} A_\alpha = \emptyset$.
(b) If $A_{\alpha_0} = X$ for some $\alpha_0 \in I$, prove that $\bigcup_{\alpha \in I} A_\alpha = X$.

P. 97, Chapter 9
At the very beginning, we should have pointed out more clearly that $A \in \mathcal{P}(S)$ if and only if $A \subseteq S$.

Here's a good problem to see if the students understand the notation:

Problem: Let A be a set. Which of the following are true?

(1) $A \in \mathcal{P}(A)$;
(2) $\emptyset \subseteq \mathcal{P}(A)$;
(3) $\emptyset = \mathcal{P}(\emptyset)$;
(4) $\{\emptyset\} = \mathcal{P}(\emptyset)$.

Students (generally speaking) often have trouble distinguishing between $\in$ and $\subseteq$. This is an opportunity to explain it again.

More Exercises

(1) Let $X = \{1, 2, 3\}$. Does $\{1\} \in X$? $\{1\} \in \mathcal{P}(X)$? $\{1, 2\} \in X$? $\{1, 2\} \in \mathcal{P}(X)$?
(2) Let $B = \{1, \{2\}\}$, true or false: $1 \in B$, $\{1\} \in B$, $\{2\} \in B$, $\{2\} \subseteq B$?

P. 103, Solution to Exercise (9.9):
The "below the line" should be replaced by "above or on."

P. 104, Additional problems for Chapter 9:

(1) Which of the following sets can be written as the Cartesian product of two subsets of $\mathbb{R}$? (Either give the two sets or explain why two such sets do not exist):
   (a) $\{(x, y) : 0 \le y \le 5\}$;
   (b) $\{(x, y) : x > y\}$;
   (c) $\{(x, y) : x^2 + y^2 = 1\}$.
(2) (a) Let X be a nonempty set. If $X \in \mathcal{P}(A \setminus B)$, must $X \in \mathcal{P}(A) \setminus \mathcal{P}(B)$?
   (b) Prove that it is never the case that $\mathcal{P}(A \setminus B) = \mathcal{P}(A) \setminus \mathcal{P}(B)$.

P.104, Problem 9.2:
It should say: "Show that $\mathcal{P}(A) \cup \mathcal{P}(B) \neq \mathcal{P}(A \cup B)$ *in general*, by exhibiting ...."

P. 105 Problem 9.12:
Add part (c): Do both implications require the sets to be nonempty? Justify your answer.

P. 106, Quote
The quote seems to be attributed to many people: Mark Twain, Blaise Pascal, Ben Franklin, Benjamin Disraeli, Stephen Leacock, Plato, Abraham Lincoln, Voltaire, St Augustine, Goethe, Cicero, Thoreau, Oscar Wilde, Samuel Johnson, Stephen Leacock, Winston Churchill, Lord Byron, among others. Numerous web writers also claim it as their own,

Most seem to cite Mark Twain, but the Pascal adherents can give a specific reference: "This letter is longer than usual simply because I could not spare the time to make it shorter. – Blaise Pascal, Lettres Provinciales (1656-1657)."

P. 111 before Exercise 10.3:
First exercise: Write the definitions of reflexive, symmetric, and transitive in symbols. Then negate the resulting definitions.

P. 112, Line -2:
Replace "$x - y = 2n$" by "$y - z = 2n$."

P. 113, Additional problem:
Define a relation on $\mathbb{R}$ by $x \sim y \leftrightarrow \exists n \in \mathbb{Z}$ such that

(a) $x, y \in [n, n+1]$;
(b) $x, y \in [n, n+1)$;
(c) $x, y \in (n, n+2)$.

For each case determine whether or not the defined relation is an equivalence relation.

P. 113, Problem 10.1
Add to problem: (k) For $x, y \in \mathbb{R}^+$ define $x \sim y$ if and only if there exists a rational number m such that $x = y^m$.

P. 115, Additional problem:
If $a, b \in \mathbb{C}$, define $a \sim b$ if and only if $a^k = b^k$ for some positive integer k. Prove that this is an equivalence relation.

P. 119, Chapter 11, additional problem:
Let $A = \{x \in \mathbb{R} : x > 0\}$ and let $B = \{x \in \mathbb{R} : x \leq 0\}$. Then $\{A, B\}$ is a partition of $\mathbb{R}$. Describe the equivalence relation associated with this partition.

P. 122 Theorem 11.4:
The following should be mentioned: Given an equivalence relation we construct a partition from the equivalence classes. This partition, in turn, induces a second equivalence relation and that one is the same as the original one. Similarly, if we start with a partition the construction leads back to the same partition.

In other words, we have a one-to-one correspondence between equivalence relations on a set and partitions of that set.

P. 123 before Exercise 11.5:
Do simple examples:

(1) Let $X = \{1, 2, 3, 4, 5\}$ and make a partition that contains three sets with $\{1, 2\}$ being one of the three.
(2) Let $X = \{1, 2, 3\}$. On $\mathcal{P}(X)$ we define $A \sim B$ if and only if $|A| = |B|$. Find the corresponding partition of $\mathcal{P}(X)$.

P. 126, Problem 11.7 lines 3 and 4
Replace by: "If you decide that it does not determine a partition, say which part(s) of the definition break down and justify your claim with an example."

P. 126, Problem 11.9:
You may want to point out to the students that the answer depends on how many sets there are. So you might ask them to see what happens when there are 2 sets, and what happens when there are more than 2 sets.

P. 147, Chapter 13:
You'll want to say something about real-valued functions and the implied domain of the function before assigning the homework.

It would have been nice to include exercises on projections and step functions in this chapter. The next edition will have some!

P. 150, line -7 and -8 (reverse signs as indicated below):
$a \geq 0$ .... $a < 0$ should be replaced by $a \leq 0$ .... $a > 0$.

P. 152, 3rd and 4th line:
Since we include the domain and codomain in the definition of a function the sentence of this line should be replace by:
Two functions $f : A \to B$ and $g : A \to B$ are equal if and only if $f(x) = g(x)$ for all $x \in A = \mathrm{dom}(f) = \mathrm{dom}(g)$.

P. 178
We've been told that we should have said more in the first paragraph of this proof (how to get from $x_1^3 - 5 = x_2^3 - 5$ to $x_1 = x_2$).

P. 184 Additional problem:
Let $H(x) = \begin{cases} x+1 & x \geq 0 \\ 1-x^2 & x < 0 \end{cases}$.

(a) Show that H is one-to-one.
(b) Let $F(x) = H(x^2 + 3x + 2)$. Is F one-to-one?

P. 202, Problem 16.4 parts (c) and (f):
Have the students prove these parts.

P. 202, Problem 16.6
Show that your guess is correct means "prove it."

P. 205 Additional problems:

(1) Show that $f(a, b) \neq (f(a), f(b))$ in general. When does $f(a, b) = (f(a), f(b))$?
(2) Let $f(x) = 9 - x^2$. Find $f((-3, 1])$ and $f^{-1}((-1, 4))$.

P. 234, Problem 18.10:
This should say, "find a different formula for f."

P. 235 Additional problem:
Let (F_n) be the Fibonacci sequence, $a = \frac{1+\sqrt{5}}{2}$, and $b = \frac{1-\sqrt{5}}{2}$. Prove that $F_n = \frac{a^n - b^n}{a-b}$ for all positive integers n.

P. 269 and 282:
The three problems below should remind students that saying that two sets are uncountable is not enough to prove that they are equivalent.

1) Add to the problems on p. 269: Prove that $(0, \infty)$ and $[0, \infty)$ are equivalent. (This is a difficult problem for students.)

2) Add to the problems on p. 282: Prove that $(0, 1)$ is equivalent to $[0, 1)$.

3) Add to the problems on p. 282: Prove that $(0, 1)$ is equivalent to $(0, 1/2) \cup [1, 2)$.

P. 282 Add a problem:
If $A \cap B = \emptyset$, then $\mathcal{P}(A \cup B) \approx \mathcal{P}(A) \times \mathcal{P}(B)$.

P. 282 Problem 22.10:
Perhaps this problem is not appropriate, since the solution requires the axiom of choice.

P. 325, Problem 25.12:
Change the second sentence to: "Prove that every *nonzero* element of $\mathbb{Z}_n$ has a reciprocal modulo n if and only if n is prime."

P. 373 line 18. Change the ciphertext to:
(12, 285, 1647)

PP. 389 ff, Index
The sets $\mathbb{R}^+, \mathbb{Q}^+$, $2\mathbb{Z}$, $3\mathbb{Z}$, and "even integers" are omitted from the index.

Thanks to those of you who sent us corrections!