Math 280, Exam 1 February 9, 2007

Name__________________________ Total /100

Write neatly and clearly, please. Show all work!!!

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(1) (20 pts.) State the contrapositive and the converse of the following. (Label each so I know which is which.)

If f is constant on the closed interval $[a, b]$, then $f'(x) = 0$ for all x in the open interval (a, b).

(2) (15 pts.) Negate the following.

Everyone who is majoring in mathematics has a friend who needs help with his homework.

(3) (20 pts.) Prove that $A \cap B = B$ if and only if $B \subseteq A$.

(4) (20 pts.) For each real number r, let $A_r = [|r|, 2|r| + 1]$. Find $\bigcap_{r \in R} A_r$ and $\bigcup_{r \in R} A_r$. Justify your answer.

(5) (15 pts.) Write symbolically assuming the universe of discourse is $\mathbf{R}$: *Every positive integer has exactly two square roots.*

(6) (10 pts.) Prove that there are no positive integer solutions to $x^2 - y^2 = 10$.

Math 280, Exam 2 March 9, 2007

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(1) Let $f(x) = \sqrt{x^2 - 4}$.
  (a) (10 pts.) Assuming the domain of f will be taken to be the largest set of real numbers for which $f(x)$ is a real number, what is the domain of f? Why? Please provide an informative sketch of your domain.
  (b) (20 pts.) Prove that the range of f is $\{y \in \mathbf{R} : y \geq 0\}$. Do this carefully.

(2) Assume all functions are well-defined.
  (a) (15 pts.) Let $f : X \to Y$ be a function. Carefully define *the inverse of* f, making sure to include all relevant requirements. Explain why your inverse function is well-defined.
  (b) (15 pts.) Let $f : \mathbf{N} \times \mathbf{N} \to \mathbf{R} \times \mathbf{R}$ be defined by $f(n, m) = (2n + m, n + 2m)$. Show that f is one-to-one.

(3) Let S be a nonempty set.
  (a) (10 pts.) Suppose S is bounded above. Define the supremum of S. You must include a definition of "upper bound" and what it means to be the "least."
  (b) (10 pts.) Find the supremum of the set $\{2 - 3/(n + 1) : n \in \mathbf{N}\}$.

(4) (20 pts.) Let $f : X \to Y$ be a function mapping X <u>onto</u> Y.
  (a) Give an example of a function f mapping $\mathbf{N}$ <u>onto</u> $\mathbf{Z}$. (Your example must be onto and well-defined, of course, but you don't have to check that it is onto or well-defined.)
  (b) Define, for each $y \in Y$ a set $A_y = \{x \in X : f(x) = y\}$. Prove that $\{A_y : y \in Y\}$ is a partition of X. Remember: f is just an arbitrary function mapping X onto Y. Not the one you defined in part (a). Make sure you justify *carefully* the partition requirements.

Math 280, Exam 3 April 20, 2007
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(1) (15 pts.) Let $x \in \mathbb{Z}^+$. Use mathematical induction to prove that $x - 1$ divides $x^k - 1$ for all positive integers k.

(2) (a) (10 pts.) Say precisely what it means for a sequence of real numbers (x_n) to diverge. (It is not enough to say that "the sequence does not converge.")

(b) (15 pts.) Let (x_n) and (y_n) be convergent sequences of real numbers. Suppose L and M are real numbers with $x_n \to L$ and $y_n \to M$. Show that $x_n + y_n \to L + M$.

(3) (15 pts.) Prove that among any five points selected inside an equilateral triangle with sides of length equal to 1, there always exists a pair at the distance not greater than .5.

(4) (15 pts.) Let $f : A \to B$ and $E \subseteq B$. Prove that $E = f(f^{-1}(E))$ if and only if $E \subseteq$ ran f. Do not quote a theorem here; show it using "element chasing."

(5) (30 pts.) Answer both of the following.

(a) Is $\{1/n : n \in Z^+\}$ finite or infinite? Why?

(b) Show that $[0, \infty)$ is equivalent to $\mathbb{R}$. (Note: This was not one of the equivalences you were given.)