A new construction of Kazhdan-Lusztig's representations of the Hecke algebra

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$v \in \mathfrak{S}_n$ is a product of generators

$$v = s_{i_1} \cdots s_{i_\ell}.$$

If $\ell = \ell(v)$ is minimal it is the length of v.

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Elements of $\mathfrak{S}_n$ are associated with permutations by the action

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- $s_1 = 2134$
- $s_3 s_1 s_2 s_1 = 3241$

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The group algebra $\mathbb{C}[\mathfrak{S}_n]$ is the $\mathbb{C}$-module with the elements of $\mathfrak{S}_n$ as generators.

Young Tableaux and Superstandard Tableaux

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Example

$\lambda = (4, 3, 1)$,

$$\begin{matrix} 1 & 3 & 6 & 7 \\ 2 & 4 & 8 & \\ 5 & & & \end{matrix} \qquad T(\lambda) = \begin{matrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & \\ 8 & & & \end{matrix}$$

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$$P(w) = \begin{matrix} 1 & 2 & 3 \\ 4 & & \end{matrix} \qquad Q(w) = \begin{matrix} 1 & 2 & 4 \\ 3 & & \end{matrix}$$

The Bruhat order on $\mathfrak{S}_n$

For $v, w \in \mathfrak{S}_n$ we say

$$v \leq w$$

if $v = s_{i_1} \cdots s_{i_\ell}$ is a subexpression of w.

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Example

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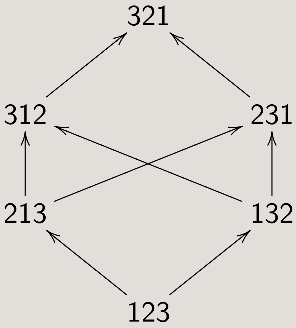

The Hecke algebra, $H_n(q)$

$\mathbb{C}[q^{\frac{1}{2}}, q^{\frac{-1}{2}}]$-algebra generated by $\{\widetilde{T}_{s_i} \mid 1 \le i \le n-1\}$ with relations

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The natural basis of $H_n(q)$ is the set of

$$\widetilde{T}_v = \widetilde{T}_{s_{i_1}} \cdots \widetilde{T}_{s_{i_{\ell(v)}}}.$$

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Notice that $H_n(1) \cong \mathbb{C}[\mathfrak{S}_n]$.

The Kazhdan-Lusztig basis of $H_n(q)$

In [Kazhdan and Lusztig, 1979] a certain basis of $H_n(q)$ is defined for each $v \in \mathfrak{S}_n$ to be

$$C'_v = \sum_{u \leq v} (q^{\frac{1}{2}})^{\ell(v)-\ell(u)} P_{u,v}(q) \widetilde{T}_u,$$

where $P_{u,v}(q)$ are the Kazhdan-Lusztig polynomials.

Return to KL immanant

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Although, $P_{u,v}(q) \in \mathbb{N}[q]$ there is no simple combinatorial description of the coefficients.

Kazhdan-Lusztig preorders on $H_n(q)$

Kazhdan-Lusztig preorders allow construction of $H_n(q)$-representations.

Right preorder

- $v \lessdot_R u$ if $a_v \neq 0$ in $C'_u \widetilde{T}_w = \sum_{z \in \mathfrak{S}_n} a_z C'_z$, for some w.

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Example

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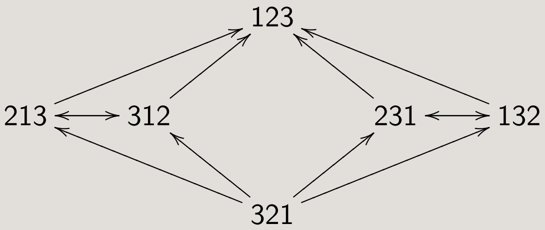

Kazhdan-Lusztig representations of $H_n(q)$

For $\lambda \vdash n$ choose a standard λ-tableau, T, and v such that $Q(v) = T$. Define

$$\begin{aligned} K^\lambda &= \mathrm{span}\{C'_u \mid Q(u) = T\} \\ &\underset{\mathrm{def}}{=} \mathrm{span}\{C'_u \mid u \leq_R v\}/\mathrm{span}\{C'_u \mid u <_R v\}, \end{aligned}$$

where $u <_R v$ means $u \leq_R v \not\leq_R u$.

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Matrix representations of $H_n(q)$ obtained by right multiplication of $\widetilde{T}_{s_i}$ on the "basis".

$$X_K^\lambda : H_n(q) \to \mathrm{GL}(d, \mathbb{C}[q^{\frac{1}{2}}, q^{\frac{-1}{2}}])$$

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Example

$$X_K^{(2,1)}(\widetilde{T}_{s_1}) = \begin{bmatrix} q^{\frac{1}{2}} & 0 \\ 1 & -q^{\frac{-1}{2}} \end{bmatrix} \qquad X_K^{(2,1)}(\widetilde{T}_{s_2}) = \begin{bmatrix} -q^{\frac{-1}{2}} & 1 \\ 0 & q^{\frac{1}{2}} \end{bmatrix}$$

Quantum polynomial ring

Define $\mathcal{A}(n;q) = \mathbb{C}[q^{\frac{1}{2}}, q^{-\frac{1}{2}}]\langle x_{1,1}, \ldots, x_{n,n}\rangle$, modulo

$$\begin{aligned}
x_{i,\ell}x_{j,k} &= x_{j,k}x_{i,\ell}, \\
x_{i,\ell}x_{i,k} &= q^{\frac{1}{2}}x_{i,k}x_{i,\ell}, \\
x_{j,k}x_{i,k} &= q^{\frac{1}{2}}x_{i,k}x_{j,k}, \\
x_{j,\ell}x_{i,k} &= x_{i,k}x_{j,\ell} + (q^{\frac{1}{2}} - q^{-\frac{1}{2}})x_{i,\ell}x_{j,k},
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for $1 \leq i < j \leq n$, $1 \leq k < \ell \leq n$.

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Convenient monomial notation: $x^{v,w} = x_{v_1,w_1} \cdots x_{v_n,w_n}$.

The immanant space and Kazhdan-Lusztig immanants

The immanant space

$\mathrm{span}\{x^{e,v} \mid v \in \mathfrak{S}_n\}$ an $n!$ dimensional subspace of $\mathcal{A}(n; q)$.

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In [Du, 1992] a dual canonical basis called Kazhdan-Lusztig immanants was defined for each $u \in \mathfrak{S}_n$

$$\mathrm{Imm}_u(x) = \sum_{v \geq u} (-q^{\frac{1}{2}})^{\ell(u)-\ell(v)} P_{w_0 u, w_0 v}(q) x^{e,v},$$

where $P_{w_0 u, w_0 v}(q)$ are the inverse Kazhdan-Lusztig polynomials.

Return to KL basis

Generalized submatrices

For n-element multisets of $[n]$ $L = (\ell(1), \dots, \ell(n))$ and $M = (m(1), \dots, m(n))$ define

$$x_{L,M} = \begin{bmatrix} x_{\ell(1),m(1)} & \cdots & x_{\ell(1),m(n)} \\ \vdots & \ddots & \vdots \\ x_{\ell(n),m(1)} & \cdots & x_{\ell(n),m(n)} \end{bmatrix}.$$

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Example

$L = (1, 1, 2)$ and $M = (2, 3, 3)$

$$x_{L,M} = \begin{bmatrix} x_{1,2} & x_{1,3} & x_{1,3} \\ x_{1,2} & x_{1,3} & x_{1,3} \\ x_{2,2} & x_{2,3} & x_{2,3} \end{bmatrix}.$$

Kazhdan-Lusztig representations of $H_n(q)$, again

For $\lambda \vdash n$ choose a standard λ-tableau, T, and v such that $Q(v) = T$. Define

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$H_n(q)$ acts on V^λ by $\widetilde{T}_u$ permuting columns of x.

$$X_V^\lambda : H_n(q) \to \mathrm{GL}(d, \mathbb{C}[q^{\frac{1}{2}}, q^{\frac{-1}{2}}])$$

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Theorem

For any $h \in H_n(q)$, $X_V^\lambda(h) = X_K^\lambda(h)$.

Vanishing of Kazhdan-Lusztig immanants

Let M an n-element multiset of $[n]$.

Theorem

If $m(i) = m(i+1)$ in M and $s_i u > u$ then $\mathrm{Imm}_u(x_{M,[n]}) = 0$.

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For $n \times n$ matrix A

$$\mu(A) = \text{row multiplicity partition of } A.$$

Dominance order of partitions, $\lambda \preceq \mu$ if

$$\sum_{i=1}^{k} \lambda_i \leq \sum_{i=1}^{k} \mu_i, \text{for all } k.$$

Theorem

If $\mathrm{sh}(u) \not\preceq \mu(x_{M,[n]})$ then $\mathrm{Imm}_u(x_{M,[n]}) = 0$.

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If $\mathrm{sh}(u) \not\preceq \mu(x_{M,[n]})$ *then* $\mathrm{Imm}_u(x_{M,[n]}) = 0$.

These results are quantum analogues to results in [Rhoades and Skandera, 2009].

Quotient-free Kazhdan-Lusztig representations of $H_n(q)$

For $\lambda \vdash n$, define the multiset $M = (1^{\lambda_1}, \dots, n^{\lambda_n})$. Define

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Matrix representations obtained by the action of $H_n(q)$ on basis of W^λ.

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For any $h \in H_n(q)$, $X_W^\lambda(h) = X_V^\lambda(h) = X_K^\lambda(h)$.

These results are $H_n(q)$ analogues to results in [B. and Skandera, 2010].

Bibliography

C. Buehrle and M. Skandera.
Relations between the Clausen and Kazhdan-Lusztig representations of the symmetric group.
To appear in *J. Pure Appl. Algebra*, 2010.

J. Du
Canonical bases for irreducible representations of quantum GL_n.
Bull. London Math. Soc., 24(4):325-334, 1992.

D. Kazhdan and G. Lusztig.
Representations of Coxeter groups and Hecke algebras.
Invent. Math., 53:165–184, 1979.

B. Rhoades and M. Skandera.
Bitableaux and the dual canonical basis of the polynomial ring.
To appear in *Adv. in Math.*, 2009.