Homework on Series

(1) Each of the series below telescopes. In each case, express the nth partial sum S_n in terms of n and determine whether the series converges or diverges.
   (a) $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)}$;
   (b) $\sum_{k=1}^{\infty} \ln(1+\frac{1}{k})$.

(2) Find $\sum_{k=0}^{\infty}(2a_k + 2^{-k})$ given that $\sum_{k=0}^{\infty} a_k = 3.57$.

(3) Evaluate $\sum_{k=0}^{\infty} \left(\frac{1}{2^k} + \frac{1}{3^k}\right)^2$.

(4) If $\sum_{k=0}^{\infty} a_k^2 = \sum_{k=0}^{\infty} b_k^2 = 4$ and $\sum_{k=0}^{\infty} a_k b_k = 3$ what is $\sum_{k=0}^{\infty}(a_k - b_k)^2$?

(5) Do the following series converge or diverge?
   (a) $\sum_{k=1}^{\infty} \frac{k^2+1}{k+5}$;
   (b) $\sum_{k=1}^{\infty}(1+1/k)^k$.

(6) From your text:
   §9.2: Problems 20, 25, 31
   §9.3: Problems 13, 29