Homework on Complex Numbers

(1) Decide whether the following statements are true or false. Justify your answers. (These are from Appendix B of your text, Exercises 37-42.)
   (a) Every nonnegative real number has a real square root.
   (b) For any complex number z, the product $z \cdot \bar{z}$ is a real number.
   (c) The square of any complex number is a real number.
   (d) If f is a polynomial, and $f(z) = i$, then $f(\bar{z}) = i$.
   (e) Every nonzero complex number z can be written in the form $z = e^w$, where w is another complex number.
   (f) If $z = x + iy$, where x and y are positive, then $z^2 = a + ib$ has a and b positive.

(2) Evaluate the following expressions and write your answers in the form $a + bi$.
   (a) $\overline{2i(\frac{1}{2} - i)}$
   (b) i^3
   (c) i^{100}

(3) Find the indicated roots, and sketch them in the complex plane.
   (a) the eighth roots of 1
   (b) the cube roots of $1 + i$

(4) Use Euler's formula to prove that $\sin 2\theta = 2 \sin\theta \cos\theta$.

(5) Let u be a complex-valued function of a real variable x. Then the indefinite integral $\int u(x)dx$ is an antiderivative of u. Use this fact to evaluate

$$\int e^{(1+i)x} dx.$$

By considering the real and imaginary parts of your answer, evaluate the two real integrals

$$\int e^x \cos x dx \quad \text{and} \quad \int e^x \sin x dx.$$

Recall that we can also evaluate these two real integrals using integration by parts. You should get the same answer both ways, although using complex numbers is easier.