Strategies for Testing Convergence and Divergence of Series
Math 202, Fall 2006

We have learned many tests to decide whether a series converges or diverges. The difficulty often lies in deciding which test to apply to a given problem. While there are no hard-and-fast rules about this, the following guidelines may be helpful.

First, try to identify the *form* of the series.

(1) If the series is of the form $\sum \frac{1}{n^p}$, then it is a p-series, which we know converges if $p > 1$ and diverges if $p \leq 1$.

(2) If it is of the form $\sum ax^{n-1}$ or $\sum ax^n$, it is a geometric series, which converges if $|x| < 1$ and diverges if $|x| \geq 1$.

(3) If it is similar to a p-series or geometric series but not exactly of this form, a comparison test might work. If a_n is a rational function of n (quotient of two polynomials) or algebraic function of n (involving roots of polynomials), then try comparing $\sum a_n$ to a p-series. Choose p by keeping only the highest powers of n in the numerator and denominator. We can only use the comparison tests if the terms of the series are all positive, but if $\sum a_n$ has some negative terms, then we may be able to use the Comparison Test on $\sum |a_n|$ and test for absolute convergence.

(4) If $\lim_{n\to\infty} a_n \neq 0$, then the Divergence Test (Thm. 9.2, part 3) should be used.

(5) If the series is alternating, try the Alternating Series Test!

(6) Series involving factorials or products such as a constant raised to the nth power are often suited to the Ratio Test. Since $|\frac{a_{n+1}}{a_n}| \to 1$ as $n \to \infty$ for all p-series (and thus all rational/algebraic functions of n), the Ratio Test should not be used for these series.

(7) If $a_n = f(n)$, where $\int_1^\infty f(x)dx$ is easily evaluated, the Integral Test may be useful.

Before applying a given test, be sure to check that the hypotheses of the test are satisfied!