Solutions to Exam 1

1. (a) $r(15) = 15e^{(-0.1)(15)} \approx \boxed{3.34695 \text{ psi/sec}}$

   (b) $\int_0^{60} 15e^{-0.1t}\,dt \approx \boxed{149.628 \text{ psi}}$

2. (a) $\text{Trap}(4) = \dfrac{\text{Right}(4)+\text{Left}(4)}{2} = \frac{1}{2}\cdot\frac{1}{4}\left[f(0)+2f(0.25)+2f(.5)+2f(.75)+f(1)\right] \approx \boxed{1.27663}$

   where $f(x) = \sqrt{1+e^{-x}}$

   (b) $\text{Mid}(4) = \frac{1}{4}\left[f(0.125)+f(0.375)+f(.625)+f(.875)\right] \approx 1.27509$

   $\text{Simp}(4) = \dfrac{2\cdot\text{Mid}(4)+\text{Trap}(4)}{3} \approx \boxed{1.2756}$

   (c) Since Trap has error proportional to n^2, using $n=40$ will be $\dfrac{\frac{1}{4^2}}{\frac{1}{40^2}} = 100$ times as accurate as using $n=4$. Simpson's rule has error proportional to n^4, so using $n=40$ will be $\dfrac{\frac{1}{4^4}}{\frac{1}{40^4}} = 10^4$ times as accurate as using $n=4$.

3. (a) $\displaystyle\int \frac{t+5}{t^2+10t+7}\,dt$

   Let $w = t^2+10t+7$, so $dw = (2t+10)dt = 2(t+5)dt$. Then

   $$\int \frac{t+5}{t^2+10t+7}\,dt = \int \frac{1}{2}\cdot\frac{dw}{w} = \frac{1}{2}\ln|w| + C = \boxed{\frac{1}{2}\ln|t^2+10t+7| + C}.$$

   (b) $\displaystyle\int \frac{y^3+y+1}{y-1}\,dy$

   We do long division:

   $$\begin{array}{r|l} & y^2+y+2 \\ \hline y-1 & y^3+0y^2+y+1 \\ & -y^3 \pm y^2 \\ \hline & \quad y^2+y \\ & \quad -y^2 \pm y \\ \hline & \qquad 2y+1 \\ & \qquad -2y \pm 2 \\ \hline & \qquad\quad 3 \end{array}$$

   So $\dfrac{y^3+y+1}{y-1} = y^2+y+2+\dfrac{3}{y-1}$, and

   $$\int \frac{y^3+y+1}{y-1}\,dy = \int \left(y^2+y+2+\frac{3}{y-1}\right)dy = \boxed{\frac{1}{3}y^3+\frac{1}{2}y^2+2y+3\ln|y-1|+C}$$

   (c) $x^2-2x-3 = (x-3)(x+1)$, and we use partial fractions

   $$\frac{x+5}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1}$$

   $x+5 = A(x+1) + B(x-3)$

   When $x=-1$, $4 = -4B \Rightarrow B = -1$.

   When $x=3$, $8 = 4A \Rightarrow A = 2$.

   So $\dfrac{x+5}{x^2-2x-3} = \dfrac{2}{x-3} - \dfrac{1}{x+1}$, and

   $$\int \frac{x+5}{x^2-2x-3}\,dx = \int\left(\frac{2}{x-3}-\frac{1}{x+1}\right)dx$$

   $$= \boxed{2\ln|x-3| - \ln|x+1| + C}$$

(d) We complete the square: $z^2-6z+10 = (z^2-6z+9)+10-9 = (z-3)^2+1$. So

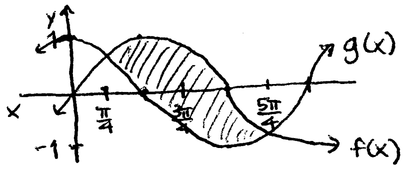
$$\int \frac{1}{z^2-6z+10}\,dz = \int \frac{1}{(z-3)^2+1}\,dz = \int \frac{1}{\tan^2\theta+1}\cdot \sec^2\theta\, d\theta = \int \frac{\sec^2\theta}{\sec^2\theta}\,d\theta = \int d\theta = \theta + C$$

Let $z-3=\tan\theta \Rightarrow \theta = \tan^{-1}(z-3)$

$dz = \sec^2\theta\, d\theta$

$$= \boxed{\tan^{-1}(z-3)+C}$$

1. Sketch the region:



$f(x)$ and $g(x)$ intersect at $\frac{\pi}{4}$ and $\frac{5\pi}{4}$, and $f(x) \geq g(x)$ on this interval

$$\text{Area of region} = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (f(x)-g(x))\,dx = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$$

$$= -\cos x \Big|_{\pi/4}^{5\pi/4} - \sin x \Big|_{\pi/4}^{5\pi/4}$$

$$= -\cos \tfrac{5\pi}{4} + \cos \tfrac{\pi}{4} - (\sin \tfrac{5\pi}{4} - \sin \tfrac{\pi}{4})$$

$$= \tfrac{\sqrt2}{2} + \tfrac{\sqrt2}{2} + \tfrac{\sqrt2}{2} + \tfrac{\sqrt2}{2} = 4\cdot\tfrac{\sqrt2}{2} = \boxed{2\sqrt2}$$

. $\int te^{at}\,dt$

Let $u=t$ $\quad dv = e^{at}dt$

$du = dt$ $\quad v = \frac{1}{a}e^{at}$

Then $\int te^{at}dt = \frac{1}{a}e^{at}\cdot t - \int \frac{1}{a}e^{at}dt$

$$= \tfrac{1}{a}te^{at} - \tfrac{1}{a}\int e^{at}dt = \tfrac{1}{a}te^{at} - \tfrac{1}{a}\cdot\tfrac{1}{a}e^{at} + C$$

$$= \boxed{\tfrac{1}{a}te^{at} - \tfrac{1}{a^2}e^{at} + C}$$

. (a) $\int 3z^2e^{-z^3}dz = -\int e^w dw = -e^w + C = -e^{-z^3}+C$

let $w = -z^3$

$dw = -3z^2dz$

So $\int_0^\infty 3z^2e^{-z^3}dz = \lim_{b\to\infty}\int_0^b 3z^2e^{-z^3}dz = \lim_{b\to\infty} -e^{-z^3}\Big|_0^b = \lim_{b\to\infty}(-e^{-b^3}+e^0)$

$$= \left(\lim_{b\to\infty} -e^{-b^3}\right)+1 = 0+1 = \boxed{1}$$

(b) $\int \frac{1}{\sqrt{4-x^2}}dx = \int \frac{2\cos\theta\, d\theta}{\sqrt{4-4\sin^2\theta}} = \int \frac{2\cos\theta}{\sqrt{4(1-\sin^2\theta)}}d\theta = \int \frac{2\cos\theta}{2\cos\theta}d\theta = \int d\theta = \theta + C = \sin^{-1}(\frac{x}{2})+C$

let $x = 2\sin\theta \Rightarrow \theta = \sin^{-1}(\frac{x}{2})$

$dx = 2\cos\theta\, d\theta$

$$\int_{-2}^{2}\frac{1}{\sqrt{4-x^2}}dx = \int_{-2}^{0}\frac{1}{\sqrt{4-x^2}}dx + \int_0^2 \frac{1}{\sqrt{4-x^2}}dx = \lim_{a\to-2^+}\int_a^0\frac{1}{\sqrt{4-x^2}}dx + \lim_{b\to2^-}\int_0^b\frac{1}{\sqrt{4-x^2}}dx$$

$$\lim_{a\to-2^+}\int_a^0 \frac{1}{\sqrt{4-x^2}}dx = \lim_{a\to-2^+}\sin^{-1}(\tfrac{x}{2})\Big|_a^0 = \lim_{a\to-2^+}(\sin^{-1}(\tfrac{0}{2})-\sin^{-1}(\tfrac{a}{2})) = \lim_{a\to-2^+}(0-\sin^{-1}(\tfrac{a}{2})) = \tfrac{\pi}{2}$$

$$\lim_{b\to2^-}\int_0^b \frac{1}{\sqrt{4-x^2}}dx = \lim_{b\to2^-}\sin^{-1}(\tfrac{x}{2})\Big|_0^b = \lim_{b\to2^-}(\sin^{-1}(\tfrac{b}{2})-\sin^{-1}(\tfrac{0}{2})) = \lim_{b\to2^-}(\sin^{-1}(\tfrac{b}{2})-0) = \tfrac{\pi}{2}$$

$$\therefore \int_{-2}^{2}\frac{1}{\sqrt{4-x^2}}dx = \tfrac{\pi}{2}+\tfrac{\pi}{2} = \boxed{\pi}$$

. $\int_1^{e^2}\frac{\ln x}{\sqrt x}dx = 2x^{1/2}\ln x\Big|_1^{e^2} - \int_1^{e^2}2x^{1/2}\cdot x^{-1}dx = 2x^{1/2}\ln x\Big|_1^{e^2} - 2\int_1^{e^2}x^{-1/2}dx$

Let $u = \ln x$ $\quad dv = x^{-1/2}dx$

$du = \frac{1}{x}dx$ $\quad v = 2x^{1/2}$

$$= 2x^{1/2}\ln x\Big|_1^{e^2} - 2[2x^{1/2}\Big|_1^{e^2}$$

$$= 2e\ln(e^2) - 2\cdot\ln 1 - 2[2e-2]$$

$$= 2e\cdot 2 - 2(2e-2) = 4e-4e+4 = \boxed{4}$$