ELEC 471
Prof. Rich Kozick

Homework 9

Date Assigned: Friday, November 5, 2010
Date Due: Friday, November 19 at 11:00 AM
(due on Friday so that Matlab help can be provided on Nov. 17)

Reading: Please read Chapter 4, Sections 4.8 - 4.10 (the material related to discrete random variables), and begin reading Chapter 3, Sections 3.1 - 3.3, on continuous random variables.

1. Please solve the following problems in the text at the end of Chapter 3.

   Problems 3.1.1, 3.1.2, 3.2.1, 3.2.2.

2. **Exam 2** will be on Wednesday, Nov. 10 at 3:00 PM. The topics for the exam will include all of Chapter 2, the beginning of Chapter 4 (sections 4.1-4.3), and Homeworks 6, 7, and 8. The exam will be open-book and open-notes, and please bring a calculator. The exam will not include a Matlab component.

3. In this problem, I would like you to perform an analysis of a digital communication system that is similar to the case that we studied in class. The only difference is that the signal S and noise N have PMFs as follows.

$$P_S(s) = \begin{cases} 0.25, & s = -1 \\ 0.75, & s = +1 \\ 0, & \text{otherwise} \end{cases} \qquad P_N(n) = \begin{cases} 0.5, & n = -1 \\ 0.25, & n = 0 \\ 0.25, & n = +1 \\ 0, & \text{otherwise} \end{cases}$$

   The random variables S and N are independent. The receiver observes the random variable $X = S + N$.

   (a) Suppose the receiver obtains the value $X = x$. Find the signal estimate $\hat{s}(x)$ that minimizes the *conditional* mean-squared error, $E[(S - \hat{s}(x))^2 \mid X = x]$. Show all of the steps in your analysis, and display $\hat{s}(x)$ as a plot versus x. I suggest that you draw plots of $P_S(s), P_N(n), P_{X,S}(x, s), P_X(x), P_{S|X}(s|x)$ as we did in class.

   (b) How would you process the signal estimates $\hat{s}(x)$ in order to recover the *binary* values $\{-1, +1\}$ of S? In other words, what "decision rule" would you use to recover the bits from X based on $\hat{s}(x)$?

   (c) For the decision rule that you developed in part b, what is the probability of a bit error for this system? Explain your reasoning.

   (d) Write a MATLAB program to simulate this system, including the decision rule developed in part b. Compare the bit error rate (BER) in your simulation with the analytical probability of a bit error computed in part c. Send your MATLAB program to Prof. Kozick by email.

   Be sure to compare the simulated and analytical BER!

4. Please answer the following questions for the joint PMF $P_{X,Y}(x,y)$ shown in the figure on page 3.

   (a) Find the marginal PMFs $P_X(x)$ and $P_Y(y)$, and plot them.

   (b) Find the mean and variance of X and Y: $\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2$.

   (c) Find the correlation between X and Y, defined as $r_{X,Y} = E[XY]$.

   (d) Find the covariance, $\text{Cov}[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = r_{X,Y} - \mu_X\mu_Y$.

   (e) Find the correlation coefficient $\rho_{X,Y}$ (see page 175 in text for definition).

   (f) Are X and Y *independent* random variables? Recall that X and Y are independent if and only if $P_{X,Y}(x,y) = P_X(x)P_Y(y)$.

   (g) Find the conditional PMFs

   $$P_{X|Y}(x|y) = P[X = x|Y = y]$$

   $$P_{Y|X}(y|x) = P[Y = y|X = x]$$

   and plot each of these versus x and y (on separate plots).

   (h) Suppose you need to produce an estimate $\hat{x}$ of X that minimizes the mean squared error $E[(X-\hat{x})^2]$. You must do this with *no information* about Y. What value should you choose for $\hat{x}$, and why? (Your answer should be a number!)

   (i) Suppose that you *observe* that the random variable Y takes on the value y (i.e., $Y = y$). We would like to incorporate the knowledge that $Y = y$ to improve our estimate of X. For each possible value of y, what is your estimate of X, denoted by $\hat{x}(y)$? Explain how to compute $\hat{x}(y)$, and present a plot of $\hat{x}(y)$ versus y.

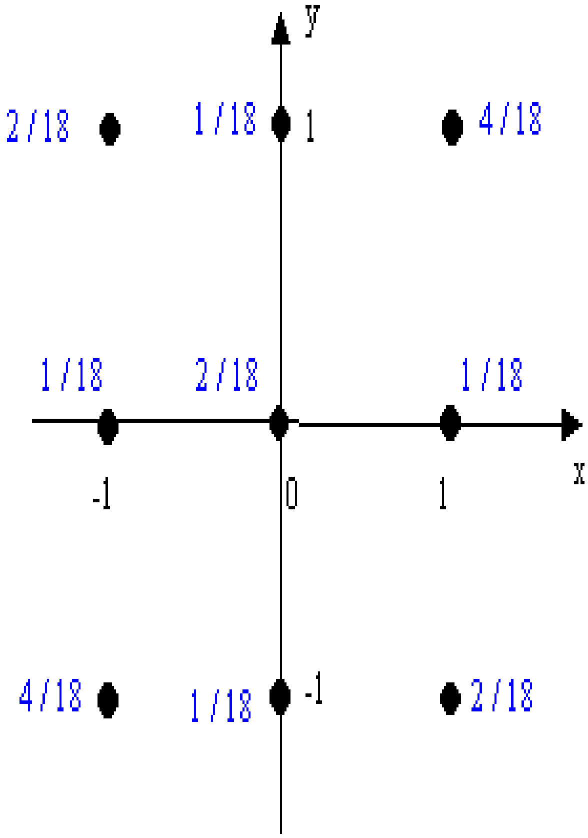
   (j) Is $\hat{x}(y)$ in part (i) different from $\hat{x}$ in part (h)? Is this reasonable based on the probability values in the joint PMF? Is this reasonable based on the value of correlation coefficient $\rho_{X,Y}$ that you computed in part (e)? That is, does the value of $\rho_{X,Y}$ lead you to expect that information about Y should be useful in predicting the value of X?

   (k) Using your answer from part (h), compute $E[(X-\hat{x})^2]$.
   Using your answer from part (i), compute
   $E[(X-\hat{x}(-1))^2|Y=-1]$, $E[(X-\hat{x}(0))^2|Y=0]$, and $E[(X-\hat{x}(1))^2|Y=1]$.
   Do these results show that we get better estimates for X when the value of Y is known? Please explain.

5. **Presentations:** The following students are asked to present their solutions on November 19 at 11:00 AM. Each pair of students will present the solution together.

   Item 3: Matt Kennedy and Dave Pike

   Item 4: Jon Schmalzle and Dylan Seeley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