ELEC 320, Fall 2007
Prof. Rich Kozick

Homework 9

Date Assigned: Wednesday, October 17, 2007
Date Due: Friday, October 19 (items 2 and 3) and Wednesday, October 24, 2007 (item 4)

Reading: Please study Chapter 3 in the Lathi text on Fourier series. We will not have time to discuss the details in Section 3.2 on correlation, but you should read it on your own. The topic is very important in signal processing applications such as time-delay estimation and digital communication.

Exam 2 will be on Monday, October 22. The topics will include the following.

- Chapter 2 on the time-domain analysis of LTIC systems (impulse response, step response, ZSR, convolution, ZIR, characteristic modes, etc.)
- Beginning concepts for Fourier series, such as items 2 and 3 on this assignment.

1. Please experiment with the Simulink demonstration of Fourier series that we used in class. You can access the demonstration from the course home page under `Class Notes and Demonstrations`.

   You do not have to submit anything for this part.

2. In the Simulink demonstration `fsgen.mdl`, the three individual sine waves that are added together are

$$f_1(t) = \sin(2\pi t), \quad f_3(t) = \sin(2\pi 3t), \quad f_5(t) = \sin(2\pi 5t).$$

   Compute the "inner products" of pairs of these sine waves over the interval $[0, 1]$. That is, compute the following integrals:

$$\int_0^1 f_1(t)f_1(t)\,dt, \quad \int_0^1 f_1(t)f_3(t)\,dt, \quad \int_0^1 f_1(t)f_5(t)\,dt$$

$$\int_0^1 f_3(t)f_3(t)\,dt, \quad \int_0^1 f_3(t)f_5(t)\,dt, \quad \int_0^1 f_5(t)f_5(t)\,dt$$

   Here are some useful identities:

$$\sin^2 a = \frac{1}{2}\left[1 - \cos(2a)\right], \quad \sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

   You may find it useful to sketch the integrand in each case.

3. What does it mean geometrically when two *vectors* have an inner (or dot) product of zero? Draw a picture of this situation in two dimensions.

4. Prove that for a continuous-time system that is *linear* and *time-invariant*, the zero-state response (ZSR) of the system to a sinusoidal input is a sine wave with the same frequency as the input wave, but a different amplitude and phase shift. Also, find an expression for the *frequency response* $H(\omega)$ of the system in terms of the *impulse response* $h(t)$.

An outline of the approach follows.

(a) Please explain why it is true that the ZSR of any linear, time-invariant (LTI) system is completely described by the impulse response $h(t)$ of the system. (Are there any LTI systems for which this is not true?) If the impulse response $h(t)$ is known, then the system output $y(t)$ due to any input $x(t)$ is given by

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\lambda) x(t-\lambda)\, d\lambda$$

(b) Now consider a particular input $x(t) = \cos(\omega_0 t)$ that is applied to a LTI system with impulse response $h(t)$. Put this $x(t)$ into the convolution integral, and look at the resulting $y(t)$. You should be able to recognize that $y(t)$ is a sine wave with the same frequency ω_0, but with a different amplitude and phase shift. The trigonometric identities at the bottom of the page will be helpful.

(c) In terms of the frequency response of the system $H(\omega)$, recall that we expect that the system output has the form

$$y(t) = |H(\omega_0)| \cos(\omega_0 t + \angle H(\omega_0)).$$

Use your result from item (b) to relate the frequency response $H(\omega)$ of the system to the impulse response $h(t)$. This provides a mathematical connection between the frequency domain and time domain descriptions of a system.

(d) You now understand the very important result that a sine wave input to a LTI system produces a sine wave output with the same frequency but different amplitude and phase shift!

Here are some useful identities:

$$\cos[\omega_0(t-\lambda)] = \cos(\omega_0 t)\cos(\omega_0 \lambda) + \sin(\omega_0 t)\sin(\omega_0 \lambda)$$

$$A\cos(\omega_0 t) - B\sin(\omega_0 t) = H\cos(\omega_0 t + \theta)$$

where $H = \sqrt{A^2 + B^2}$ and $\theta = \arctan(B/A)$.