EXAMPLE 2.5 For the square signal $g(t)$ shown in Fig. 2.17 find the component in $g(t)$ of the form $\sin t$. In other words, approximate $g(t)$ in terms of $\sin t$:

$$g(t) \simeq c \sin t \qquad 0 \le t \le 2\pi$$

so that the energy of the error signal is minimum.

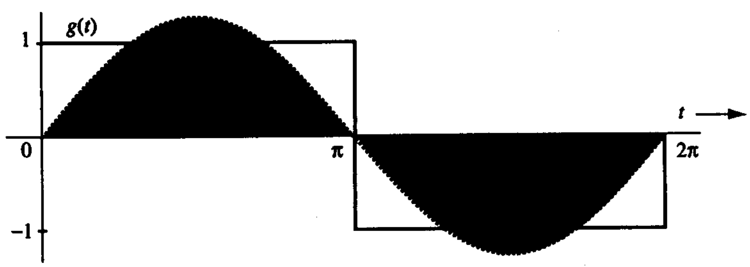

Figure 2.17 Approximation of a square signal in terms of a single sinusoid.

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* For complex signals the definition is modified as in Eq. (2.40).

In this case,

$$x(t) = \sin t \qquad \text{and} \qquad E_x = \int_0^{2\pi} \sin^2 t \, dt = \pi$$

From Eq. (2.31), we find

$$c = \frac{1}{\pi} \int_0^{2\pi} g(t) \sin t \, dt = \frac{1}{\pi} \left[\int_0^{\pi} \sin t \, dt + \int_{\pi}^{2\pi} - \sin t \, dt \right] = \frac{4}{\pi} \quad (2.33)$$

Therefore,

$$g(t) \simeq \frac{4}{\pi} \sin t \qquad (2.34)$$

represents the best approximation of $g(t)$ by the function $\sin t$, which will minimize the error energy. This sinusoidal component of $g(t)$ is shown shaded in Fig. 2.17. By analogy with vectors, we say that the square function $g(t)$ shown in Fig. 2.17 has a component of signal $\sin t$ and that the magnitude of this component is $4/\pi$.

conditions, and hence possesses a convergent Fourier series. Thus, a physical possibility of a periodic waveform is a valid and sufficient condition for the existence of a convergent series.

EXAMPLE 2.8 Find the compact trigonometric Fourier series for the periodic square wave $w(t)$ shown in Fig. 2.22a, and sketch its amplitude and phase spectra.

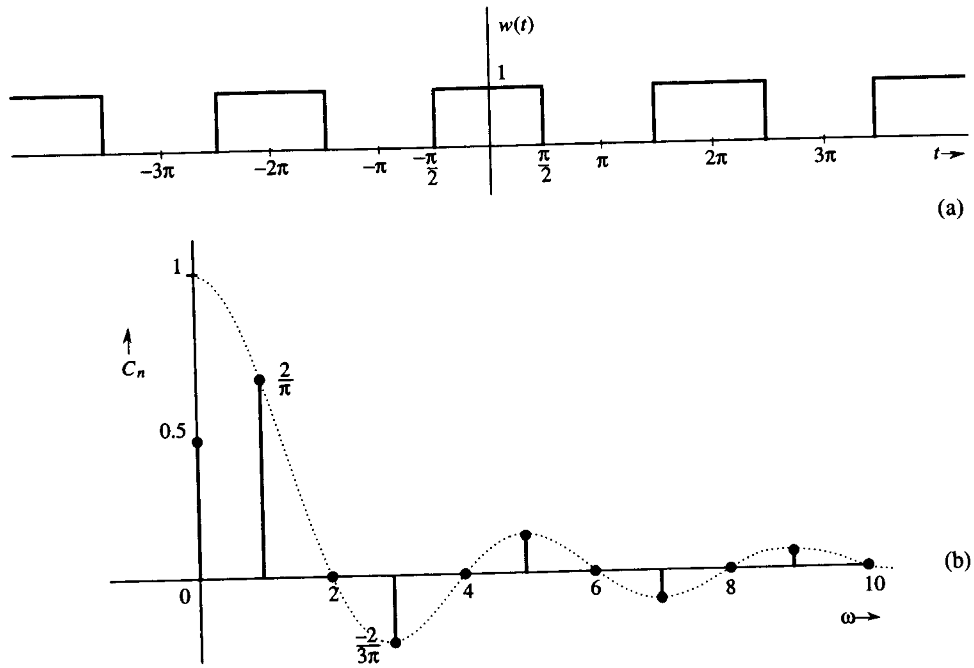

Figure 2.22 Square pulse periodic signal and its Fourier spectra.

The Fourier series is

$$w(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

where

$$a_0 = \frac{1}{T_0} \int_{T_0} w(t)\, dt$$

In the preceding equation, we may integrate $w(t)$ over any interval of duration T_0. Figure 2.22a shows that the best choice for a region of integration is from $-T_0/2$ to $T_0/2$. Because $w(t) = 1$ only over $(-T_0/4,\ T_0/4)$ and $w(t) = 0$ over the remaining segment,

$$a_0 = \frac{1}{T_0} \int_{-T_0/4}^{T_0/4} dt = \frac{1}{2} \tag{2.74a}$$

We could have found a_0, the average value of $w(t)$, to be 1/2 merely by inspection of $w(t)$ in Fig. 2.22a. Also,