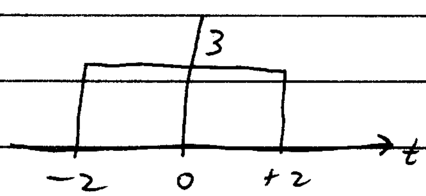
Fourier Transform Operations

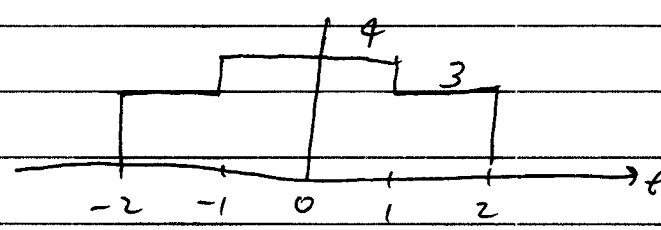
Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(x)\, dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

$$f(t) \cdot \cos(\omega_0 t) \Longleftrightarrow \frac{1}{2} F(\omega - \omega_0) + \frac{1}{2} F(\omega + \omega_0)$$

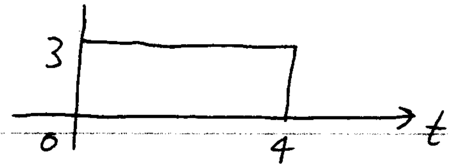
Rayleigh's energy thm.: $\displaystyle\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega$

<u>Linearity</u>: Find Fourier transforms of





Time Shift :



F. T. $\Rightarrow$

Time scaling : Play a tape at ½ speed and 2x speed.

Time convolution: Apply to LTI systems

$$y(t) = f(t) * h(t) \Leftrightarrow Y(\omega) = F(\omega) \cdot H(\omega)$$

Ex : Cosine pulse with finite duration.

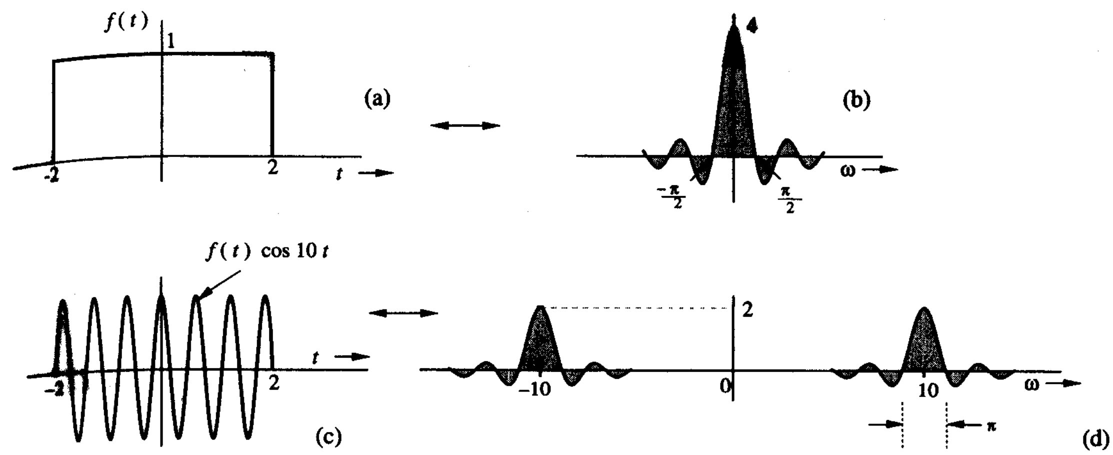


Fig. 4.24 An example of spectral shifting by amplitude modulation.

Why ?