Step and Natural Responses for RC and RL Circuits

For RC and RL circuits, the step and natural responses are obtained by solving a first-order, linear differential equation with constant coefficients and constant "input" (or forcing function),

$$\frac{dx}{dt} + ax(t) = b,$$

where a and b are constants, and the initial value $x(t_0)$ is known. The solution is

$$x(t) = \left[x(t_0) - \frac{b}{a}\right] e^{-a(t-t_0)} + \frac{b}{a}, \quad \text{for } t > t_0.$$

The function $x(t)$ may be either a voltage or a current in the circuit. Note that the time constant is

$$\tau = \frac{1}{a}.$$

It is useful to write the equation in words:

$$x(t) = [\text{initial value} - \text{final value}] \cdot \exp\left(-\frac{t - \text{starting time}}{\text{time constant}}\right) + \text{final value},$$

for $t >$ the starting time. We can often obtain the "initial value," "final value," and "time constant" using circuit analysis, and then we can immediately write the solution from the previous equation.

Important note: The "initial value" refers to time $t = t_0^+$, i.e., the instant *after* a change occurs in the circuit (such as switches opening or closing).