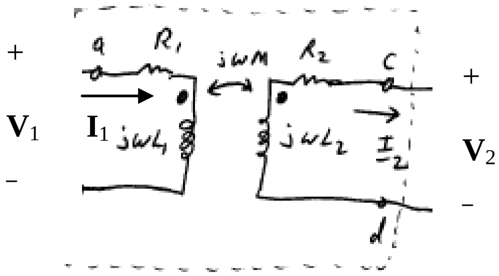

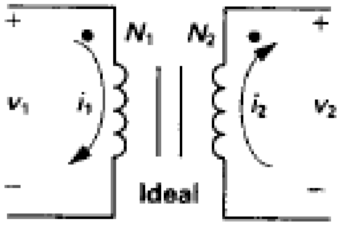

Consider linear and ideal transformers attached to Circuit 1 and Circuit 2.

$$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 - (j\omega M)\mathbf{I}_2$$
$$\mathbf{V}_2 = (j\omega M)\mathbf{I}_1 - (R_2 + j\omega L_2)\mathbf{I}_2$$
$$\mathbf{I}_2 = \frac{j\omega M}{j\omega L_2 + R_2 + \mathbf{Z}_L}\mathbf{I}_1$$

$$? \rightarrow ? \quad \frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}$$
$$N_1\,\mathbf{I}_1 = N_2\,\mathbf{I}_2$$

Substitute $M = k\sqrt{L_1 L_2}$ with $0 \le k \le 1$ and pull out factors in the $\mathbf{V}_1$ and $\mathbf{V}_2$ equations:

$$\mathbf{V}_1 = j\omega L_1\left[\left(1+\frac{R_1}{j\omega L_1}\right)\mathbf{I}_1 - k\sqrt{\frac{L_2}{L_1}}\,\mathbf{I}_2\right]$$
$$\mathbf{V}_2 = j\omega L_1\sqrt{\frac{L_2}{L_1}}\left[k\,\mathbf{I}_1 - \sqrt{\frac{L_2}{L_1}}\left(1+\frac{R_2}{j\omega L_2}\right)\mathbf{I}_2\right]$$
$$\mathbf{I}_2 = \frac{k}{1+\frac{R_2+\mathbf{Z}_L}{j\omega L_2}}\sqrt{\frac{L_1}{L_2}}\,\mathbf{I}_1$$

$\Rightarrow$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \sqrt{\frac{L_1}{L_2}}\;\frac{\left(1+\frac{R_1}{j\omega L_1}\right)\mathbf{I}_1 - k\sqrt{\frac{L_2}{L_1}}\,\mathbf{I}_2}{k\,\mathbf{I}_1 - \sqrt{\frac{L_2}{L_1}}\left(1+\frac{R_2}{j\omega L_2}\right)\mathbf{I}_2}$$
$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \sqrt{\frac{L_2}{L_1}}\;\frac{1+\frac{R_2+\mathbf{Z}_L}{j\omega L_2}}{k}$$

What must occur to achieve the *ideal* transformer relations?

1) $k = 1,\quad (\omega L_1) >> R_1,\quad (\omega L_2) >> R_2,$ and $\sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2}$ for voltage ratios.
2) In addition, $(\omega L_2) >> |\mathbf{Z}_L|$ is needed for current ratios.

Comparisons between linear transformer (more accurate) and ideal (when conditions are satisfied):
Open-circuit load: $\mathbf{Z}_L = \infty, \mathbf{I}_2 = 0$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \sqrt{\frac{L_1}{L_2}}\;\frac{\left(1+\frac{R_1}{j\omega L_1}\right)}{k} \quad \text{so} \quad \left|\frac{\mathbf{V}_2}{\mathbf{V}_1}\right| = \sqrt{\frac{L_2}{L_1}}\;\frac{k}{\left|1+\frac{R_1}{j\omega L_1}\right|} < \frac{N_2}{N_1}$$

Any finite load: $|\mathbf{Z}_L| < \infty$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \sqrt{\frac{L_2}{L_1}}\;\frac{1+\frac{R_2+\mathbf{Z}_L}{j\omega L_2}}{k} \quad \text{so} \quad \left|\frac{\mathbf{I}_1}{\mathbf{I}_2}\right| = \sqrt{\frac{L_2}{L_1}}\;\frac{\left|1+\frac{R_2+\mathbf{Z}_L}{j\omega L_2}\right|}{k} > \frac{N_2}{N_1}$$