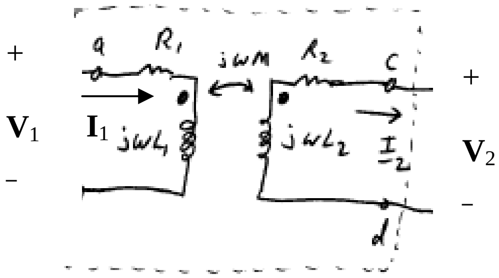
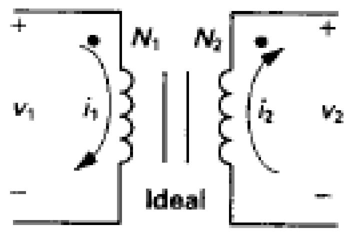
Linear Transformer → Ideal Transformer





Consider linear and ideal transformers attached to Circuit 1 and Circuit 2.

$$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$
$$-\mathbf{V}_2 = -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2$$

$$? \to ? \quad \frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}$$
$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

$$L_1 = N_1^2\mathcal{P}_{12}\left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right) = N_1^2\mathcal{P}_{12}g_1, \quad g_1 \ge 1, \qquad L_2 = N_2^2\mathcal{P}_{12}\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = N_2^2\mathcal{P}_{12}g_2, \quad g_2 \ge 1$$

$$M = N_1N_2\mathcal{P}_{12} \qquad \frac{M^2}{L_1L_2} = k^2 = \frac{1}{g_1g_2} \le 1$$

Perfect coupling : $k^2 = 1 \Rightarrow g_1 = g_2 = 1$

Substitute in linear transformer equations for L_1, L_2, M :

$$\mathbf{V}_1 = (R_1 + j\omega N_1^2\mathcal{P}_{12}g_1)\mathbf{I}_1 - (j\omega N_1N_2\mathcal{P}_{12})\mathbf{I}_2 \Rightarrow \left(\frac{N_2}{N_1}\right)\mathbf{V}_1 = \left(\frac{N_2}{N_1}R_1 + j\omega N_1N_2\mathcal{P}_{12}g_1\right)\mathbf{I}_1 - (j\omega N_2^2\mathcal{P}_{12})\mathbf{I}_2 \to \mathbf{V}_2$$

$$\mathbf{V}_2 = (j\omega N_1N_2\mathcal{P}_{12})\mathbf{I}_1 - (R_2 + j\omega N_2^2\mathcal{P}_{12}g_2)\mathbf{I}_2 \Rightarrow \left(\frac{N_1}{N_2}\right)\mathbf{V}_2 = (j\omega N_1^2\mathcal{P}_{12})\mathbf{I}_1 - \left(\frac{N_1}{N_2}R_2 + j\omega N_1N_2\mathcal{P}_{12}g_2\right)\mathbf{I}_2 \to \mathbf{V}_1$$

What must occur to achieve the limits on the right side?

$$\left[\omega N_1N_2\mathcal{P}_{12}g_1 >> \frac{N_2}{N_1}R_1 \text{ or } \omega L_1 >> R_1\right] \text{ and } \left[\omega N_1N_2\mathcal{P}_{12}g_2 >> \frac{N_1}{N_2}R_2 \text{ or } \omega L_2 >> R_2\right] \text{and } [g_1 = g_2 = 1]$$

Now solve the linear transformer equations for the currents:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{R_1 + j\omega N_1^2\mathcal{P}_{12}g_1} + \frac{j\omega N_1N_2\mathcal{P}_{12}}{R_1 + j\omega N_1^2\mathcal{P}_{12}g_1}\mathbf{I}_2 \Rightarrow \frac{N_1}{N_2}\mathbf{I}_1 = \frac{\frac{N_1}{N_2}\mathbf{V}_1}{R_1 + j\omega N_1^2\mathcal{P}_{12}g_1} + \frac{j\omega N_1^2\mathcal{P}_{12}}{R_1 + j\omega N_1^2\mathcal{P}_{12}g_1}\mathbf{I}_2 \to \mathbf{I}_2$$

$$\mathbf{I}_2 = \frac{j\omega N_1N_2\mathcal{P}_{12}}{R_2 + j\omega N_2^2\mathcal{P}_{12}g_2}\mathbf{I}_1 - \frac{\mathbf{V}_2}{R_2 + j\omega N_2^2\mathcal{P}_{12}g_2} \Rightarrow \frac{N_2}{N_1}\mathbf{I}_2 = \frac{j\omega N_2^2\mathcal{P}_{12}}{R_2 + j\omega N_2^2\mathcal{P}_{12}g_2}\mathbf{I}_1 - \frac{\frac{N_2}{N_1}\mathbf{V}_2}{R_2 + j\omega N_2^2\mathcal{P}_{12}g_2} \to \mathbf{I}_1$$

Also need $\left|\frac{\mathbf{V}_1}{R_1 + j\omega L_1}\right| << |\mathbf{I}_1|$ and $\left|\frac{\mathbf{V}_2}{R_2 + j\omega L_2}\right| << |\mathbf{I}_2|$, so currents are due to coupling (only).

What is needed in transformer construction to achieve these limits?

Problem 9.79: Find Z_{ab}, then repeat with the dot on one coil reversed.

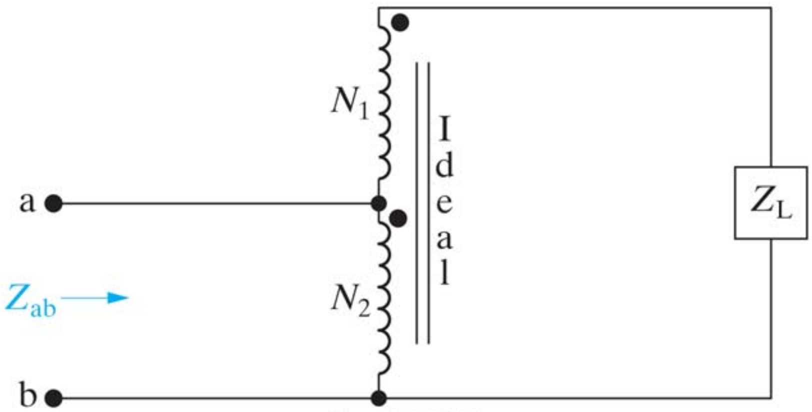


Figure: 09-58-56P9.79