Manifestation of classical phase in a single spontaneously emitted photon

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Abstract: We note a manifestation of the classical phase of a traveling wave in a multimode quantized field. We study spontaneous emission, scattering, and re-absorption by two-level atoms in a one-dimensional optical cavity, and observe classical phase information in the complex quantum amplitude for re-absorption of the scattered field.

The concept of phase for a quantum radiation field has been investigated from the early days of quantum mechanics. (For a review, see reference [1].) Most previous work has focussed on defining an appropriate phase operator for states of a single-mode field. In this paper we investigate a one-photon field that is the result of spontaneous emission; this field state is a linear combination of many singly-occupied field modes. The classical phase and amplitude information does not appear in any quantity directly related to a field operator, but rather it appears in the complex quantum amplitude for transitions to states in which the spontaneously emitted photon has been subsequently re-absorbed.

We study a fully quantized analog of the the simple classical treatment of scattering given by Feynman [2]. We consider various sets of two-level atoms at fixed positions within a one-dimensional multimode optical cavity of length L. The specific results presented here are for two atoms: atom 1 located at $x_1 = L/2$ and atom 2 located at $x_2 = 3L/4$. Atom 1 begins in the excited state and spontaneously emits a photon, which subsequently scatters from atom 2. The quantum amplitude for the re-excitation of atom 1 by the scattered photon exhibits the same phase and magnitude information contained in a classical field scattered by a harmonic oscillator dipole.

Our Hamiltonian assumes both the electric dipole and rotating-wave approximations [3]; both of the atoms have space-dependent couplings to all of the quantized standing-wave cavity modes. The state of the system is written as

$$|\psi(t)\rangle = c_1(t)|e,g;\emptyset\rangle + c_2(t)|g,e;\emptyset\rangle + \sum_k d_k(t)|g,g;k\rangle, \tag{1}$$

where $|e,g;\emptyset\rangle$ represents a state with atom 1 excited, atom 2 in the ground state, and no photons in the field, $|g,e;\emptyset\rangle$ represents a similar state, except that it is atom 2 which is excited, and $|g,g;k\rangle$ represents a state with both atoms in the ground state and a single photon in the k^{th} field mode. The atoms have resonance frequencies $\omega^{(1)}_{atom}$ and $\omega^{(2)}_{atom}$; the central frequency of the field emitted by atom 1 is detuned from the frequency of atom 2 by $\omega^{(1)}_{atom} - \omega^{(2)}_{atom}$. We determine the time evolution of the system by calculating eigenstates and energies of the *total* Hamiltonian, and then projecting the initial state of the system onto the eigenstates, whose evolution is known [4]. Our results were calculated using 299 field modes, resulting in a 301×301 Hamiltonian matrix.

Fig. 1 illustrates the magnitude of the amplitude for atom 1 to be found in the excited state, $|c_1(t)|$, for several values of the detuning of the atoms. This amplitude decays exponentially from its initial value of 1. (The atom-field couplings were chosen

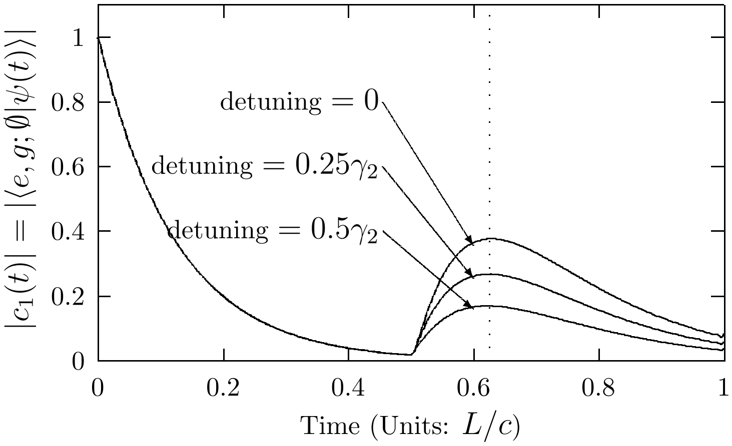

Fig. 1. Amplitude for initially excited atom to be found in the excited state.

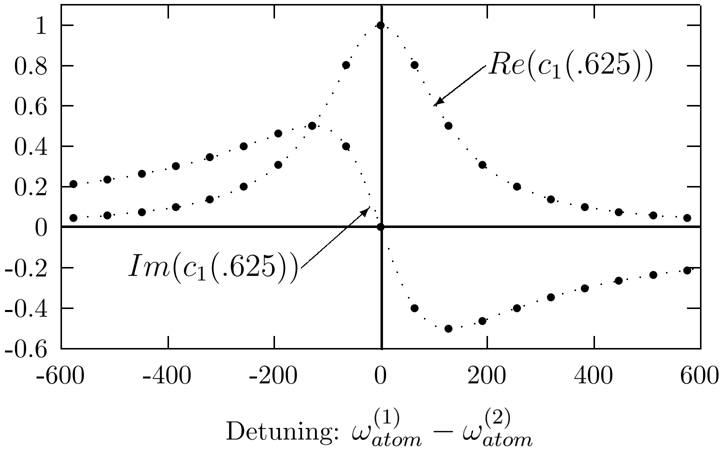

Fig. 2. Real and imaginary parts of the complex amplitude for the initially excited atom to be in the excited state ($c_1 = \langle e, g; \emptyset|\psi\rangle$) at time $t = 0.625$. (The data have been normalized to the zero-detuning amplitude.)

so that the atoms have probability decay constants $\gamma_1 = 16$ and $\gamma_2 = 256$ in the units of the figures.) At time $t = 0.5L/c$ the field reradiated by atom 2 first returns to atom 1, causing an abrupt increase in $|c_1(t)|$. As the relative detuning between the two atoms increases, the effect of the scattered photon on atom 1 decreases. The phase of the complex number c_1 also changes. Fig. 2 illustrates the real and imaginary parts of c_1 at time $t = .625$ (when the effect of the scattered field from atom 2 is the largest) as a function of the relative detuning of the atoms. The points are the results of our calculations, and the dotted lines give the well-known theoretical prediction for the real and imaginary parts of the complex amplitude of the *classical* field scattered from a harmonic oscillator dipole having the same energy decay constant as atom 2.

References and links

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2. R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, (Addison-Wesley, Reading, MA, 1963), vol. I, Chapters 31 and 32.
3. P. Meystre and M. Sargent III, *Elements of Quantum Optics*, (Springer, Berlin, 1999)
4. M. Ligare and S. Becker, Am. J. Phys., **63**, 788-796 (1995)