Two-Photon Interference?

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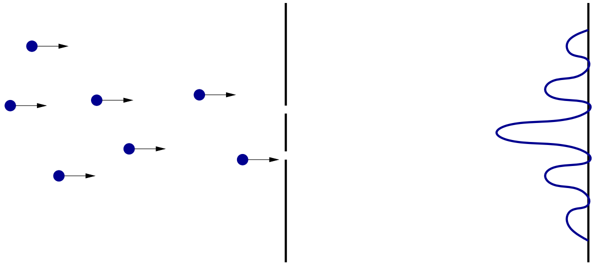
Two-slit Interference



- Detection as particles.
- Distribution of detections as if waves.
- At low intensity, only one “particle” in apparatus at a time.

Interference

Dirac:

> *"Each photon then interferes only with itself. Interference between two different photons never occurs."*

Wave-Particle Duality

Photons: Waves or Particles?

Points to remember:

- Photons are massless.
- Inherently relativistic.
- Non-relativistic Schrödinger equation doesn't tell us anything about photons; there isn't a wavefunction $\psi(x)$ for a photon.
- Light is described by a relativistic quantum field theory.

Wave-Particle Duality

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Better questions:

- What can we measure?
- What are the differences between the predictions of a classical field theory and the predictions of a quantum field theory?

Measurements

Intensity (Measured at single point)

Classical: Proportional to square of a measurable field strength
Quantum: Rate of detection of photons

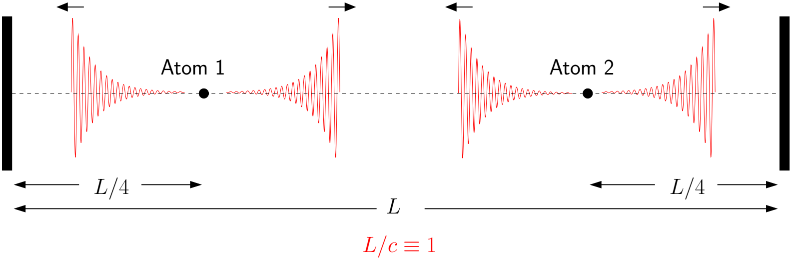
Sensitivity to phase of fields (interference)?

Intensity Correlation (Measured at two points)

Classical: Proportional to product of squares of field strengths
Quantum: Rate of detections of two photons (joint probability)

Sensitivity to phase of fields (interference)?

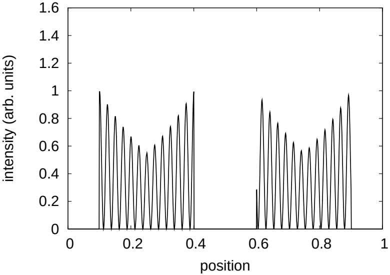
Simple Model



- One-dimension.
- Single Polarization.
- Atoms
  - Classical: Random-phase dipole oscillators
  - Quantum: Two-level atoms

Classical Field Intensity at $t = 0.15$

Instantaneous



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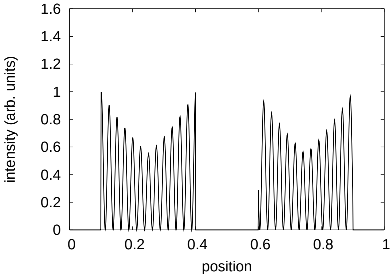
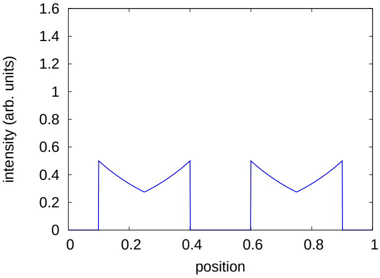
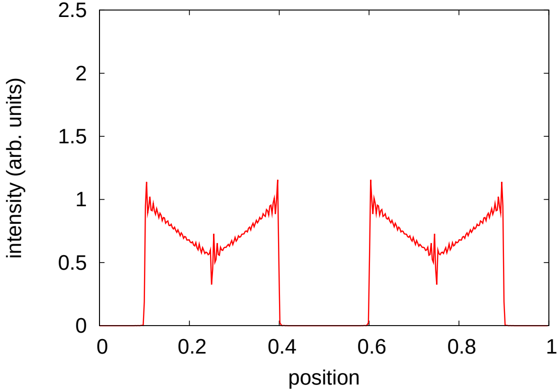


Averaged over period and random phases



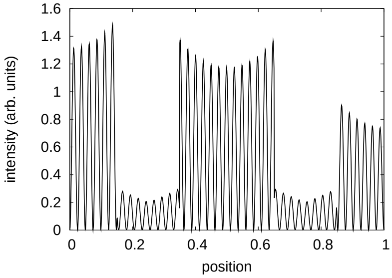
Quantum Field "Intensity" at $t = 0.15$

$$\langle\psi| : \hat{E}(x)\hat{E}(x) : |\psi\rangle$$

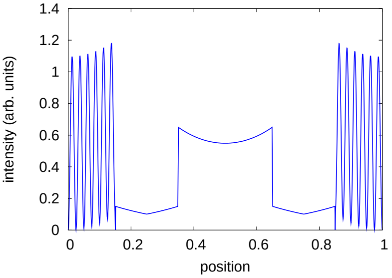


Classical Field Intensity at $t = 0.4$

Instantaneous

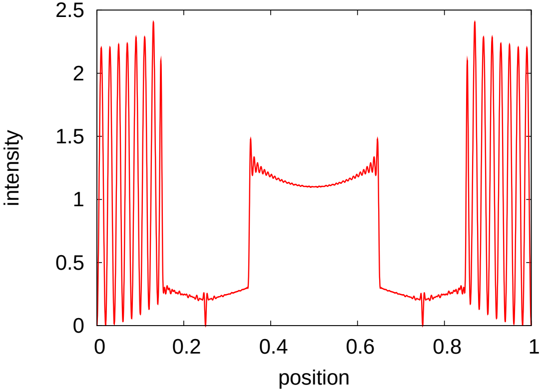


Averaged over period and random phases



Quantum Field "Intensity" at $t = 0.4$

$$\langle\psi| : \hat{E}(x)\hat{E}(x) : |\psi\rangle$$



Dramatic Pause

Intensity Correlation

Two detectors are better than one!

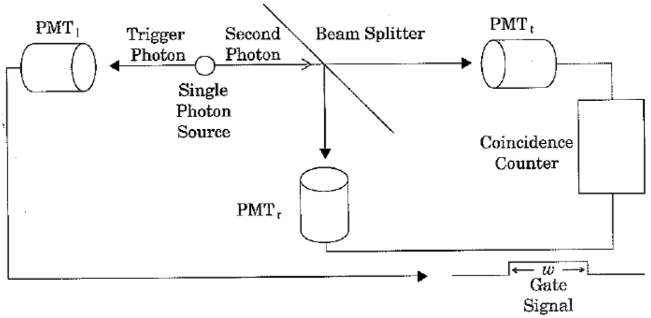


Figure 2–5 *Anticoincidence Experiment of Aspect and Co-workers.*[5] The trigger photon from the single-photon source is detected: this alerts the two detectors PMT$_t$ and PMT$_r$ to expect a photon sometime during the brief “gate period” w.

Intensity Correlation

Classical Field:

$$I(x_1)I(x_2) \longrightarrow \langle \overline{I}(x_1)\overline{I}(x_2)\rangle_{\phi_1,\phi_2}$$

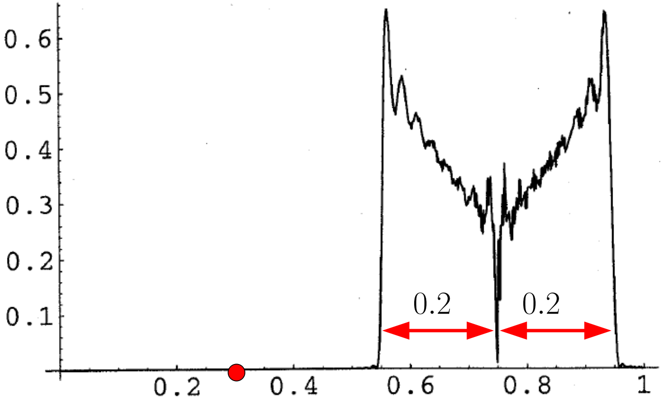
Quantum Field:

(Prob. of Detecting Photon at x_1)$\times$(Prob. of Detecting Photon at x_2)

Quantum Intensity Correlation Function

$$\langle \hat{E}(x_1)\hat{E}(x_2)\hat{E}(x_2)\hat{E}(x_1)\rangle$$

$x_1 = 0.3$ $t = 0.2$

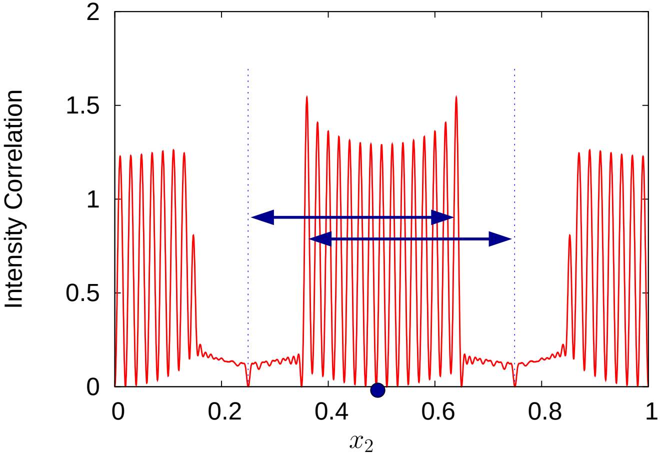


x_2

Quantum Intensity Correlation Function

$$\langle \hat{E}(x_1)\hat{E}(x_2)\hat{E}(x_2)\hat{E}(x_1)\rangle$$

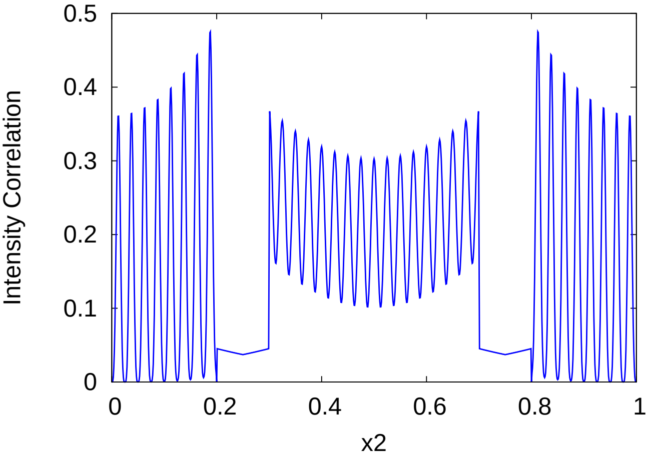
$x_1 = 0.5$ $\quad$ $t = 0.4$



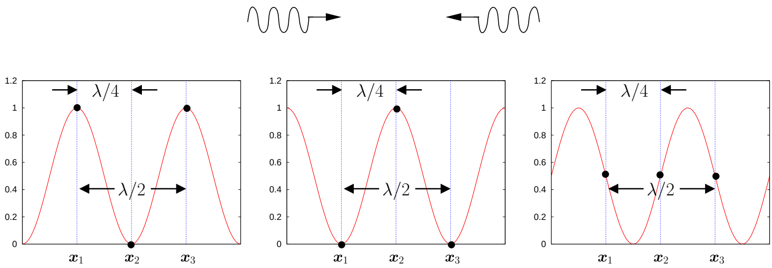
Classical Intensity Correlation Function

$$\langle \overline{I}(x_1)\overline{I}(x_2)\rangle_{\phi_1,\phi_2}$$

$x_1 = 0.5$ $t = 0.45$

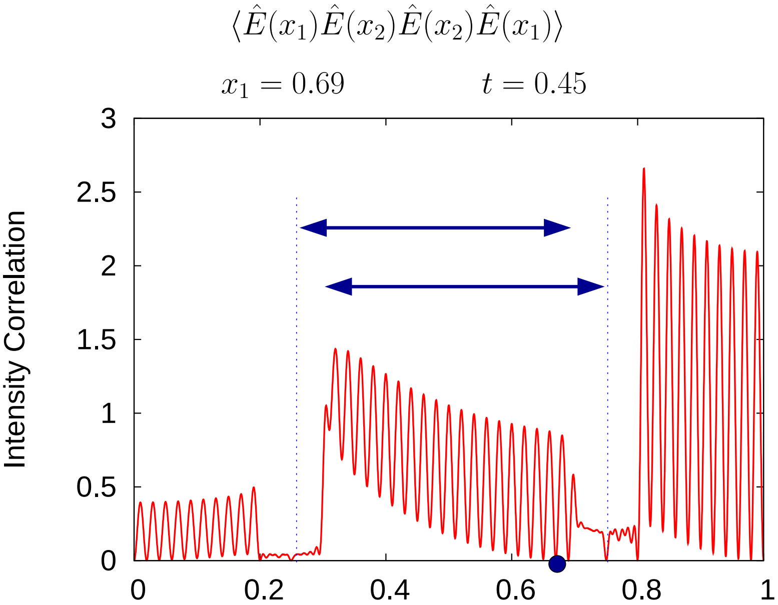


Interference in Classical Correlation (Hand Waving)



$\Delta\phi$	$I(x_1)$	$I(x_2)$	$I(x_3)$	$I(x_1) \times I(x_2)$	$I(x_1) \times I(x_3)$
0	1	0	1	0	1
π	0	1	0	0	0
$\pi/2$	1/2	1/2	1/2	1/4	1/4
Avg.				1/12	5/12

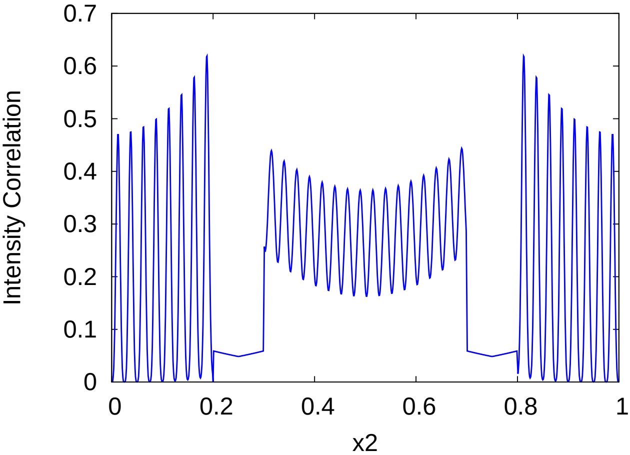
Quantum Intensity Correlation Function



Classical Intensity Correlation Function

$$\langle \overline{I}(x_1)\overline{I}(x_2)\rangle_{\phi_1,\phi_2}$$

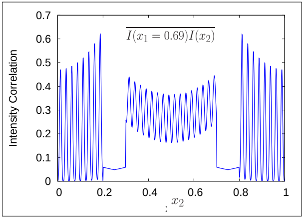
$x_1 = 0.69$ $\qquad$ $t = 0.45$



Correlation: Quantum vs. Classical

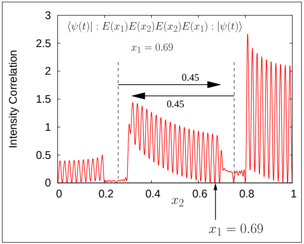
Classical Field:

$$|E_L(0.69)+E_R(0.69)|^2 \times |E_L(x_2)+E_R(x_2)|^2$$



Quantum Field:

$$|\mathcal{A}_L(0.69)\mathcal{A}_R(x_2) + \mathcal{A}_L(x_2)\mathcal{A}_R(0.69)|^2$$



Simplest Field Theory

Model Features:

- "Modes of the universe" (1-D); Quantized standing wave modes
- Multiple modes (201) $\to$ quasi-continuum
- Spontaneous emission via interaction with multiple empty modes.
- Schrödinger picture.
- $\longrightarrow$ "Localized" photons.

Simplest Field Theory

Basis States:

$\|e\,e;0\rangle$:	both atoms excited, no photons
$\|e\,g;1_k\rangle$:	atom 1 excited, atom 2 in g.s., 1 photon (mode k)
$\|g\,e;1_k\rangle$:	atom 1 in g.s. atom 2 excited, 1 photon (mode k)
$\|g\,g;1_k,1_{k'}\rangle$:	both atoms in g.s., 2 photons in distinct modes
$\|g\,g;2_k\rangle$:	both atoms in g.s., 2 photons in same mode

Field Operators:

$$\hat{E}(x) = \sum_k C_k \left(a_k + a_k^\dagger\right) \sin\left[(k_0 + k)\frac{\pi x}{L}\right]$$

Simplest Field Theory

Initial State: $|\psi(0)\rangle = |e\,e;0\rangle$

Time-Dependent State:

$$|\psi(t)\rangle = a(t)|e\,e;0\rangle + \sum_k b_{1k}(t)|e\,g;1_k\rangle + \sum_k b_{2k}(t)|g\,e;1_k\rangle + \sum_{k,k'<k} c_{k,k'}(t)|g\,g;1_k,1_{k'}\rangle + \sum_k d_k(t)|g\,g;2_k\rangle$$

Hamiltonian: Two-level atoms, RWA, multimode.

$$\begin{aligned} H &= H_{\text{atoms}} + H_{\text{field}} + H_{\text{interaction}} \\ &= \hbar\omega_{eg}^{(1)}\sigma_3^{(1)} + \hbar\omega_{eg}^{(2)}\sigma_3^{(2)} + \sum_k \hbar\omega_k\left(a_k^\dagger a_k + \frac{1}{2}\right) \\ &\quad + \sum_k \hbar\left(\Omega_1\sigma_+^{(1)}a_k + \Omega_1^*\sigma_-^{(1)}a_k^\dagger\right)\sin\left[(k_0+k)\frac{\pi x_1}{L}\right] \\ &\quad + \sum_k \hbar\left(\Omega_2\sigma_+^{(2)}a_k + \Omega_2^*\sigma_-^{(2)}a_k^\dagger\right)\sin\left[(k_0+k)\frac{\pi x_2}{L}\right], \end{aligned}$$

Idiosyncratic (but simple) Dynamics Calculation

Project initial state onto energy eigenstates:

$$\begin{aligned} |\psi(0)\rangle &= |e, e; 0\rangle \\ &= \sum_q |E_q\rangle\langle E_q|e, e; 0\rangle \end{aligned}$$

Use known time evolution of eigenstates:

$$|\psi(t)\rangle = \sum_q e^{-iE_q t/\hbar}|E_q\rangle\langle E_q|e, e; 0\rangle.$$

Project onto state of interest, e.g.:

$$\begin{aligned} c_{kk'}(t) &= \langle g, g; 1_k, 1_{k'}|\psi(t)\rangle \\ &= \sum_q e^{-iE_q t}\langle g, g; 1_k, 1_{k'}|E_q\rangle\langle E_q|e, e; 0\rangle \end{aligned}$$

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Conclusions

- Photons are strange (non-classical).
- Photons do retain some aspects of classical attributes (phase, relative phase).
- The nature of photons can be probed via non-local correlations.
- It's amplitudes that interfere, not fields.

Thanks to:

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